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Scheduling Long-Distance Transport Operations under Labor Regulations: A Hybrid Optimization Approach

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Abstract

This work solves a case study of a transportation company that aims to plan the maximum number of jobs requested by customers that must be performed over a week in order to minimize the cost of assigning such jobs to vehicles. In this specific problem every customer pays for a job which is composed by a set of different operations (representing loading and unloading), which in turn has associated a location and a time windows. Once a job is assigned to a vehicle, all operations must be completed by the same vehicle within the specific time window. We have addressed this problem using two approaches. The first approach addresses the problem globally by formulating its constraints and solving it using the Hexaly solver (a black-box optimizer), while the second approach makes a partition of the problem to make the best decision at each step using a matheuristic. To compare the pros and cons of each approach, different scenarios have been considered.

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1. Introduction

In a globalized economy, the efficiency and effectiveness of transportation networks are crucial for sustaining competitive advantage across industries. To address these challenges, optimization techniques have become indispensable tools, enabling improved transportation system performance and reduced operational cost. Among these techniques, the assignment, scheduling, and vehicle routing problems stand out as three of the most extensively studied combinatorial optimization problems, each playing a critical role in enhancing transportation network efficiency.

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The problem of interest in this work focuses on the optimization of the job-vehicle assignment of a transportation company operating in Spain. In general, the transportation company case study attempts to find the optimal allocation of resources (vehicles) to a sequence of jobs requested by customers over a week, aiming to satisfy a set of constraints and optimize specific performance criteria. Time constraints could be considered such as a predefined time window for the jobs and resource constraints could be imposed since the number of resources is limited.

Classical and foundational contributions in the field, such as Laporte's review of fifty years of vehicle routing research Laporte (2009), highlight the evolution of exact and heuristic methods for the VRP and related scheduling problems. Furthermore, for problems involving large-scale planning under complex resource and time constraints, column generation techniques have proven particularly effective, as presented in the seminal work by Desaulniers, Desrosiers, and Solomon Desaulniers et al. (2006).

Moving to more recent developments, we refer the reader to Lv et al. (2023), which addresses a problem closely related to ours by modeling ferry vehicle scheduling at airports as an unrelated parallel machine scheduling problem, solved with a variable neighborhood search algorithm to improve punctuality and balance vehicle workload. Another relevant work is Davatgari et al. (2024), where heuristic and metaheuristic frameworks are developed for scheduling electric bus fleets from a single depot, incorporating charging requirements and next-day operability to reduce emissions and operational costs. Finally, Can Atasagun and İsmail Karaoğlan (2024) proposes a mixed-integer programming model and a memetic algorithm to optimize the integrated production and distribution of perishable products from multiple heterogeneous facilities, considering limited product shelf life and capacity constraints. While these represent some of the most similar studies, none fully address the comprehensive operational requirements of long-distance transportation companies. To the best of our knowledge, no existing research simultaneously incorporates heterogeneous fleets, sequential multi-operation jobs with strict time windows, and detailed labor regulations within a long-term transportation planning framework.

This work is structured in the following way. Section 2 details the transportation company case study. Section 3 explains the two methodologies that we have been implemented to tackle the case study. Section 4 compares and discusses the results. Finally, Section 5 concludes the work and includes a future research line.

2. Problem definition

The transport company operates across the Iberian Peninsula and plans its operations on a monthly basis. To achieve this, the planning process is divided into weekly periods and solved using an Iterative Decision-Making (IDM) or Rolling Horizon approach, Sahin et al. (2013), with the planning horizon updated every 10 minutes. The main objective is to assign customer-requested jobs to available vehicles while ensuring compliance with the following requirements.

There is a one-to-one relationship between drivers and vehicles, meaning that each vehicle is always operated by the same driver. The company owns a heterogeneous fleet of vehicles so, each vehicle has specific characteristics in terms of capacity and has associated a different depot. The depot determines the location where every vehicle must start and end the route during week. Furthermore, all vehicles (and thus their drivers) must respect the labor regulations and also, rest periods for vehicles named *breaks*. In this study, a simplification of the Spanish legal framework regarding driving and rest periods has been applied. Specifically, it is assumed that each vehicle must observe a daily rest period of 11 consecutive hours. Furthermore, a vehicle is not permitted to drive for more than 9 hours in a single day unless a break is taken. In such cases, the total daily working time, including breaks, must not exceed 15 hours.

Each customer requests the transportation company to perform a job consisting in a number of operations. Every job consists of operations that have associated a location, a service time and a time window. These operations may consist of loading or unloading the vehicle. Jobs may be accepted or refused by the transportation company. The operations of a job must be completed in a specific sequence. Uncompleted jobs will incur in a penalty, in this case 10000 monetary units since this can be viewed as the money lost. A vehicle cannot handle two jobs simultaneously. The travel cost (distance and time) between every pair of operation locations is deterministic and known beforehand. In our specific problem, distance and time are correlated measures since we assume that vehicles travel at 75 km/h and that there is a cost per km traveled, in this case 1 monetary unit. Therefore, we can translate this travel cost into monetary units, length units or time units as appropriate.

The objective function is to minimize total cost, which include both the cost per kilometer traveled and penalties for jobs that are not completed.

3. Algorithmic Proposal

This section presents two different methodologies that have been implemented in order to tackle the problem of interest in this work. Both approaches handle the problem from a different perspective.

In the first approach, we address the comprehensive problem including in the problem by means of a formulation that includes all constraints. This approach is implemented using Hexaly¹. See Subsection 3.1. The second approach proposes to solve the problem by means of matheuristic algorithm. This methodology is done by using a partition of the week. In each partition of the week, jobs are assigned to vehicles recursively. Then, an exact algorithm determines the best option to schedule the vehicles that will be used in the whole week. See Subsection 3.2.

3.1. Comprehensive approach

We name this proposal as the *Comprehensive approach* since we model the entire problem by incorporating all particular constraints considered by the transportation company to obtain a feasible solution. Therefore, with this comprehensive perspective we can address the whole problem without incorporating any simplification of the problem. The implementation leverages the Hexaly solver, utilizing its decision variable structures, such as list variables to define the list of operations for each vehicle and interval variables to represent specific service time Inc. (2024). We have opted by Hexaly because of its advanced capabilities which are able to model features representing realistic situations. Specifically, we make use of interval decision variables, they are used to model the time interval of an operation. They are declared using the built-in function defining an integer minimum value and an integer maximum value, where the limits are integers representing the minimum start and maximum end of the decision. We also use list decision variables that creates an ordered collection of integer within a domain. The introduction of these special structures considerably reduces the number of decision variables, which speeds up the optimization process by allowing more iterations to be performed in the same time.

In addition, during the modeling of the problem using Hexaly, these decision variables can be used as parameters when defining the constraints, which facilitates the development and understanding of the model.

3.2. Matheuristic approach

For this approach we have developed a heuristic greedy algorithm to generate initial feasible solutions combined with a binary linear programming to optimize specific parts of our optimization problem while maintaining all constraints.

The sets and parameters used in this methodology can be seen in Table 1

The decision variables used are denoted by $\{\gamma_i^v\}_{v \in V, i \in R^v}$, which take a value of 1 if sequence i is selected for vehicle v , and 0 otherwise. These binary variables indicate whether a particular sequence of compatible jobs is assigned to a specific vehicle. Their role is central to the optimization process, as they determine which job sequences are ultimately included in the final solution, ensuring that all operational and assignment constraints are satisfied.

The first step of this algorithm aims to assign potential sequences of jobs to vehicles, in this way we defined $R^v \forall v \in V$. This process is addressed modeling a graph, where nodes represent jobs and edges represent compatible jobs. It is considered that two jobs are compatible if time window constraints and operating range constraints are satisfied. To that end, the proposed algorithm consists of an explicit enumeration of all compatible jobs and vehicles. Under this approach vehicles incrementally add jobs to their schedules, always ensuring operational rules, such as time windows and rest period regulation. As this exhaustive enumeration has an exponential growth as function of the size of the input problem, we propose to make a partition of the problem. Therefore, to deal with this issue, we divide the week into two parts. Then, and compatible jobs are assigned to vehicles just for the first part of week. Then, these

¹ Hexaly is a global optimization solver that offers nonlinear and set-oriented modeling APIs.

Category	Symbol	Description	Domain / Notes
Sets			
Sets	V	Vehicles	–
Sets	J	Jobs/Orders	–
Sets	R^v	Possible job sequences for each vehicle v	$\forall v \in V$
Sets	R_j^v	Possible job sequences for v that contain job j	$\forall v \in V, j \in J$
Parameters			
Parameters	d_i^v	Total distance of sequence i for vehicle v	$\forall i \in R^v, \forall v \in V$
Parameters	o_i^v	Number of orders in sequence i for vehicle v	$\forall i \in R^v, \forall v \in V$
Parameters	N_o	Total number of orders	$N_o = J $
Parameters	P_o	Penalty for unplanned orders	–

Table 1: Sets and parameters of the BLP

assignments serve as the basis for generating the schedules for the second part of the week. This phasing significantly reduces computational complexity, allowing the algorithm to manage a more manageable problem partition. The second step of the algorithm is in charge of deciding which jobs are scheduled in every vehicle for the first part of the week, because up to this point we have an enumeration of all compatible jobs to vehicles, but we still need to optimize this process. This is done in an optimal way by using the binary linear programming model defined in Equations (1)–(4). The objective function (1) represents the total cost of assigning job sequences to vehicles, combining the total distance traveled by the fleet with a penalty for unassigned orders; thus, it aims to minimize the adjusted cost of each selected sequence—considering both the distance and the number of included jobs—along with a fixed penalty for each unassigned job. Regarding the constraints, expression (2) ensures that each vehicle is assigned at most one sequence, thereby avoiding overlapping or conflicting assignments. Similarly, constraint (3) guarantees that each job appears in at most one sequence across all vehicles, ensuring exclusive assignment. Lastly, constraint (3) also defines the binary nature of the decision variables, enforcing that each sequence is either selected or not for a given vehicle.

Once the route for the first part of the week is determined, we repeat the process for the last part of the week. Note that, the decision taken in the first part of the week will condition the decisions taken in the second part of the week and that is why at the beginning of the section we mention that this approach is greedy. The logical structure of the proposed algorithm is visually illustrated in Figure 1.

$$\begin{aligned} \min C &= \sum_{v \in V} \sum_{i \in R^v} \gamma_i^v d_i^v + P_o \left(N_o - \sum_{v \in V} \sum_{i \in R^v} \gamma_i^v o_i^v \right) \\ &= \sum_{v \in V} \sum_{i \in R^v} \gamma_i^v (d_i^v - P_o o_i^v) + P_o N_o \end{aligned} \quad (1)$$

$$\sum_{i \in R^v} \gamma_i^v \leq 1 \quad \forall v \in V \quad (2)$$

$$\sum_{v \in V} \sum_{i \in R_j^v} \gamma_i^v \leq 1 \quad \forall j \in J \quad (3)$$

$$\gamma_i^v \in \{0, 1\} \quad \forall i \in R^v, \forall v \in V \quad (4)$$

The matheuristic approach is significant as it integrates the advantages of mathematical optimization and heuristic algorithms. Although precise mathematical models ensure optimal or near-optimal solutions, they frequently encounter computational complexity as the problem size increases. Conversely, heuristics offer rapid, practical solutions, albeit perhaps lacking in accuracy. The matheuristic approach adeptly combines a greedy constructive phase with a mathematical model to effectively navigate the solution space, achieving an optimal balance between solution quality and processing efficiency. Moreover, since it is a solution that involves a constructive phase, by adjusting this phase it can

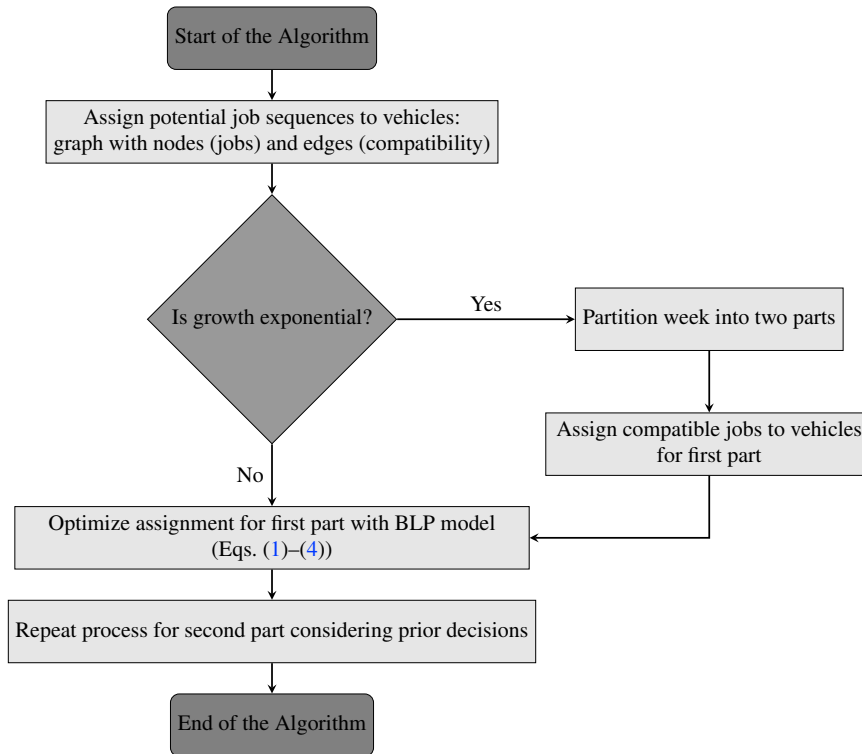


Fig. 1: Flowchart of the proposed matheuristics algorithm

be applied to a wide variety of optimization problems beyond transportation, such as vehicle routing with multiple constraints, scheduling, and resource allocation in manufacturing and services, where efficient and scalable solution methods are essential.

4. Computational results

In this section, we present and discuss the results obtained from executing the two approaches described in Section 3. Both approaches have been executed in a computer equipped with an AMD Ryzen 9 5950x 16-core processor (32 threads) and 128GB of RAM. The first one is implemented using Hexaly 13.0.2. This section is structured in the following way. First of all, we will describe the instances that will be solved. Finally, different scenarios are presented to perform a comparison between the two approaches.

Due to the proprietary nature of this project and the confidentiality policies of the transportation company, the source code and the specific instances used in this work cannot be shared. Nevertheless, all methodologies, algorithms, and approaches have been described in detail to ensure the understanding of the research within the limits of the disclosed information.

We would like to remark that to solve every instance using the algorithms, we allow to the solver a time limit of 30 minutes (1800 seconds). By using these configurations, we aim to balance computational efficiency and solution quality in both approaches, while providing a foundation for comparative analysis between methodologies.

16 real-world instances were provided by the transportation company. These original 16 real-world instances have different characteristics that can be seen in Table 2. Operations have to be scheduled to satisfy a hard time window in a specific day. We say that such operations have a *fixed appointment*. However, other operations are allowed to be scheduled considering soft time windows specifically, at any time within the opening hours during the appointment day, that is, it has associated a wider time window. Furthermore, some operations have a *flexibility level* regarding the

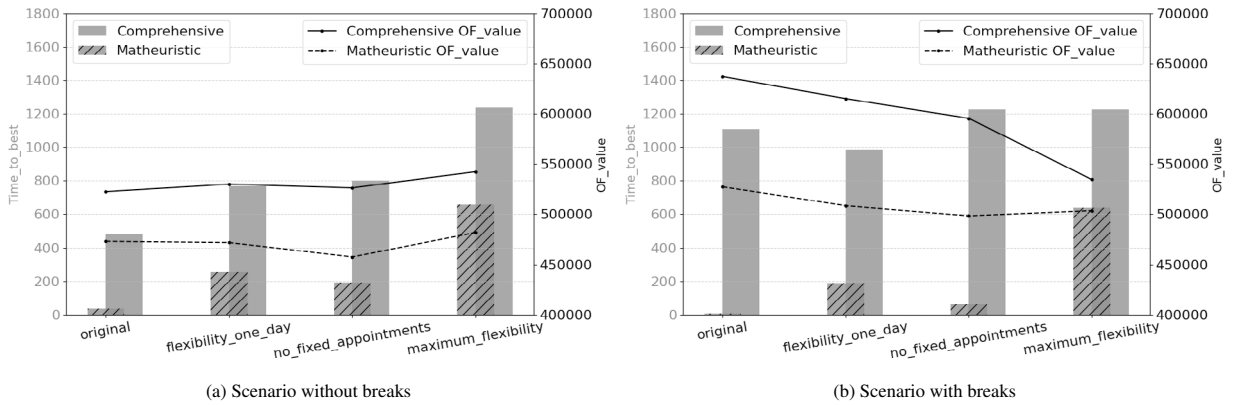


Fig. 2: Comparison plots for Scenarios

day to be scheduled. Therefore, we name *null flexibility*, if the operation may be scheduled in a specific day, and *a day flexibility* if the operation may be scheduled one day before or one day after the original appointment.

To testbed a richer set of instances we have modified the characteristics of the original set of 16 instances, resulting in a total of 64 instances. Then, the first subset is the original set of instances. The second set assumes that all operations have dynamic flexibility, that is, they could be scheduled one day before and after the original appointment. The third set considers that all operations have no fixed appointment while preserving the original flexibility level, that is, regarding the day to be scheduled. And finally, the last set removes the fixed appointment and the flexibility level, so here we are allowing the maximum flexibility to schedule the operations.

4.1. Description of the instances

Instance	#Jobs	#Operations
1	79	158
2	76	155
3	80	160
4	70	146
5	69	143
6	87	175
7	115	235
8	124	257
9	133	270
10	147	306
11	143	300
12	130	277
13	16	32
14	132	280
15	139	280
16	157	334

Table 2: Instances characteristic

To solve the instances, the computational results consider two different scenarios: considering and not considering the labor regulation of breaks, see Table 3 and Figure 2a and Table 4 and Figure 2b, respectively. We include both scenarios in the experiments to analyze the impact of this constraint.

Figure 2a represents the scenario without breaks and we can observe that greater flexibility in the instances requires longer execution times for their resolution using any of the two methodologies than using the original instances.

	Original		A day flexibility		No fixed appointment		Maximum flexibility	
	Comp.	Math.	Comp.	Math.	Comp.	Math.	Comp.	Math.
Number of Jobs planned								
Avg	55.69	61.06	56.06	62.44	57.62	65.00	58.19	64.75
Std	13.45	18.95	13.78	20.23	13.41	19.12	14.47	17.94
Min	14.00	13.00	14.00	12.00	15.00	15.00	15.00	12.00
Max	73.00	95.00	74.00	102.00	75.00	98.00	76.00	90.00
Number of Vehicles used								
Avg	13.50	12.81	13.06	12.69	13.00	11.19	12.38	12.56
Std	2.80	2.51	2.74	2.75	2.58	2.54	2.70	2.85
Min	4.00	5.00	4.00	4.00	4.00	4.00	4.00	4.00
Max	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00
Total cost (in thousand of euros)								
Avg	522.41	473.01	530.00	471.73	526.21	457.37	542.40	481.85
Std	284.47	229.05	294.28	231.95	288.62	234.39	301.94	265.84
Min	24.31	35.61	24.31	44.31	14.85	24.25	14.85	44.55
Max	952.27	824.64	1011.54	830.15	902.84	842.28	931.60	910.28
Time to obtained the best solution (in seconds)								
Avg	480.00	36.94	766.56	252.50	800.44	189.75	1237.50	657.62
Std	518.23	61.25	693.35	275.40	517.05	162.94	558.27	255.65
Min	1.00	1.00	1.00	0.00	7.00	1.00	121.00	1.00
Max	1713.00	223.00	1688.00	702.00	1688.00	495.00	1788.00	975.00

Table 3: Descriptive statistics of the Scenario without breaks

	Original		A day flexibility		No fixed appointment		Maximum flexibility	
	Comp.	Math.	Comp.	Math.	Comp.	Math.	Comp.	Math.
Number of Jobs planned								
Avg	44.06	55.44	47.62	58.62	50.81	60.75	59.56	62.56
Std	10.18	15.82	13.08	16.96	11.70	16.22	14.06	16.44
Min	11.00	11.00	11.00	12.00	12.00	10.00	14.00	12.00
Max	53.00	73.00	66.00	83.00	64.00	80.00	74.00	84.00
Number of Vehicles used								
Avg	13.88	13.38	13.94	13.81	13.88	12.31	14.38	13.31
Std	2.73	2.73	2.79	2.54	2.83	2.57	2.50	2.77
Min	4.00	4.00	4.00	5.00	4.00	4.00	5.00	4.00
Max	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00
Total cost (in thousand of euros)								
Avg	637.22	527.24	615.18	508.47	595.49	498.29	534.57	503.58
Std	319.03	260.97	297.08	247.06	310.89	258.24	302.72	293.98
Min	52.94	52.83	52.94	43.97	44.18	63.04	24.48	44.45
Max	1062.53	938.09	1101.41	909.56	932.78	861.34	1008.43	1026.58
Time to obtained the best solution (in seconds)								
Avg	1107.88	5.94	988.06	187.50	1228.38	63.56	1226.00	637.12
Std	546.75	9.96	528.47	273.59	515.86	91.39	527.87	256.17
Min	3.00	1.00	2.00	1.00	235.00	1.00	108.00	1.00
Max	1784.00	37.00	1619.00	710.00	1786.00	327.00	1711.00	971.00

Table 4: Descriptive statistics of Scenario with break

Additionally, we note that, on average, the matheuristic approach achieves better results than the comprehensive algorithm in significantly shorter execution times in all the set of instances. In this first scenario, focusing only on the value of the objective function of minimizing the total costs, the comprehensive algorithm outperforms only in 19 out of the 64 instances.

Looking in more detail at Table 3, we also observe a greater stability in these two metrics. Regarding the fleet, the matheuristic approach uses fewer vehicles and completes a higher average number of jobs. However, it shows a higher standard deviation across instances which indicates that the results are more dispersed in relation to the average results.

Figure 2b shows the trend seen so far, but in a less marked form. In this case, despite having more flexibility, the constraints regarding the breaks means that we have a more restrictive and rigid operation in both methodologies. Therefore, the execution time is less sensitive to flexibility, especially in the comprehensive methodology. In terms of objective function, the matheuristic approach outperforms in the majority of the instances.

Looking now in more detail at Table 4, we observe a slight increase in the objective function with respect to the scenario without breaks.

5. Conclusions and Future Research

In this work, we have solved a transportation company case study, which aims to schedule jobs into vehicles in order to minimize the total cost incurred by the operation. Furthermore, our model considers all constraints imposed by the company and other legal regulations.

We have considered two approaches to solve the case study. On the one hand, a comprehensive approach in which all constraints are included without considering any simplification to speed it up the implementation. On the other hand, a matheuristic approach is proposed in which a greedy construction phase and a mathematical model are combined as the matheuristics do. The matheuristic approach demonstrates greater overall efficiency, excelling in both resource utilization and computational time. It consistently finds optimal solutions faster than the comprehensive approach, making it a strong contender in scenarios where rapid problem-solving and minimized resource usage are crucial.

Our work addresses a gap in the literature by simultaneously modeling heterogeneous fleets, sequential multi-operation jobs with strict time windows, and detailed labor regulations within a long-term transportation planning framework—constraints that are commonly encountered in real-world long-haul logistics. The methodological novelty lies in the design of an integrated optimization approach that combines the flexibility of greedy constructive method with the structure and accuracy of mathematical programming. By embedding a constructive layer that guides the solver's search space, we enhance both the practical applicability and the computational efficiency of the solution process. This hybrid framework allows us to tackle complex, large-scale scheduling problems in a way that pure mathematical or heuristic methods alone often cannot.

For future research we would like to explore the incorporation of intelligent assumptions to reduce the number of generated sequences, potentially eliminating the need for this arbitrary cap.

References

- Can Atasagun, G., İsmail Karaođlan, 2024. Integrated production and outbound distribution scheduling problem with multiple facilities/vehicles and perishable items. *Applied Soft Computing* 166, 112144.
- Davatgari, A., Cokyasar, T., Verbas, O., Mohammadian, A.K., 2024. Heuristic solutions to the single depot electric vehicle scheduling problem with next day operability constraints. *Transportation Research Part C: Emerging Technologies* 163, 104656.
- Desaulniers, G., Desrosiers, J., Solomon, M.M., 2006. Column generation. volume 5. Springer Science & Business Media.
- Inc., H., 2024. Hxoperator — hexaly 13.0 documentation. Accessed: 2024-12-10.
- Laporte, G., 2009. Fifty years of vehicle routing. *Transportation science* 43, 408–416.
- Lv, L., Deng, Z., Shao, C., Shen, W., 2023. A variable neighborhood search algorithm for airport ferry vehicle scheduling problem. *Transportation Research Part C: Emerging Technologies* 154, 104262.
- Sahin, F., Narayanan, A., Robinson, E.P., 2013. Rolling horizon planning in supply chains: review, implications and directions for future research. *International Journal of Production Research* 51, 5413 – 5436. doi:[10.1080/00207543.2013.775523](https://doi.org/10.1080/00207543.2013.775523).